Week 5
MATH 34B
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14. The speed of car A after t minutes is $8t\ m/s$.

    How long will it take the car to travel $\frac{100}{6}$ meters?

$v(t) = 8t$

$p(t) = 4t^2$

$4t^2 = \frac{100}{6}$

$\Rightarrow t^2 = \frac{100}{24}$

$\Rightarrow t = \sqrt{\frac{100}{24}}$

16. How quickly a leaf grows is proportional how big [ie the surface area] the leaf is. If the area of the leaf grows from $2cm^2$ to $3cm^2$ in 3 days, how long will it take for the leaf's area to increase to $5cm^2$?

See week 4. solutions

36. The population of a country Dnalgne is 100 million in 1997 and increasing at a rate of 0.6 million per year. The average annual income of a person in Dnalgne during 1997 was 24000 dollars per year and increasing at a rate of 500 dollars per year.

How quickly was the total income of the entire population rising in 1997?

See week 4 solutions

27. An artery has a circular cross section of radius 4 millimeters. The speed at which blood flows along the artery fluctuates as the heart beats. The speed after t seconds is $30 + 5\sin(2\pi t)$ meters per second. What volume of blood passes along the artery in one second?

See Week 4 Solutions

47. The volume of a sphere of radius R is $V(R) = 4R^3/3$. If the radius is increased by ΔR, what is the increase in volume to the first order?

Typically, to differentiate,

we do $\frac{dV}{dt} = 3 \cdot \frac{4\pi R^2}{3} \cdot \frac{dR}{dt}$

But we replace $\frac{dR}{dt}$ with ΔR (as it represents the change of R).

So, we get

$4\pi R^2 \Delta R$

50. Find a linear approximation to the function $f(x) = e^{x/500}$ for the range $0 < x < 100$. Do this by making the linear approximation equal to the function at the end points x=0 and x=100. Find the percent error in the approximation when (a) $x = 25$ and (b) $x = 50$.

See Week 4 solutions.

43. What are the local max/min for $y = e^x \sin(x)$ between $x = 0$ and $x = 2\pi$?

$y' = e^x \cos(x) + e^x \sin(x)$

set $= 0$

$\Rightarrow e^x \cos(x) + e^x \sin(x) = 0$

As $e^x \neq 0$ for any x, we can divide by e^x.

$\Rightarrow \cos(x) + \sin(x) = 0$

$\cos(x) = -\sin(x)$

$1 = \frac{-\sin x}{\cos x} = -\tan(x)$.

$\Rightarrow x = 3\pi/4,\ 7\pi/4$.

$y'' = e^x \cos x + e^x(-\sin x) + e^x \sin x + e^x \cos x$

$= 2 e^x \cos x$.

$y''(\frac{3\pi}{4}) = 2e^{3\pi/4} \cos(3\pi/4) < 0$

$y''(\frac{7\pi}{4}) = 2e^{7\pi/4} \cos(7\pi/4) > 0$

So, $y(\frac{3\pi}{4})$ local max,

$y(\frac{7\pi}{4})$ local min.

12. Assume the amount of pollution entering the world's oceans grows exponentially. At the start of 1900 suppose that the instantaneous rate at which pollution enters the oceans is 10^5 tons per year, and that the amount doubles every 10 years thereafter. Express the rate that pollution enters, in units of tons per year, at a time t years after 1900.

Find an integral which expresses the total amount of pollution which has entered during the period 1900 to 1990.

$P(0) = 10^5$

$P(t) = 10^5 2^{t/10}$ (doubles every ten years)

To get total amount of pollution, we evaluate

$$\int_0^{90} 10^5 2^{t/10}\, dt$$

$$= \int_0^{90} 10^5 e^{\frac{t}{10}\ln 2}\, dt$$

$$= 10^5 e^{\frac{t}{10}\ln 2} \cdot \frac{10}{\ln 2} \Big|_0^{90}$$

$$= 10^5 \cdot \frac{10}{\ln 2} \left(e^{\frac{90}{10}\ln 2} - 1\right)$$